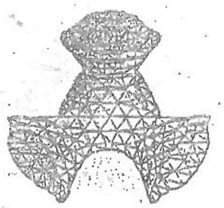
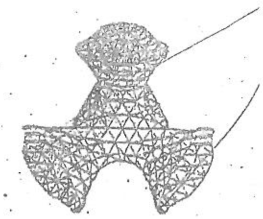
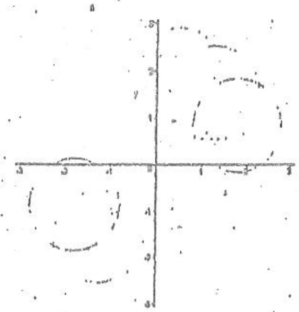
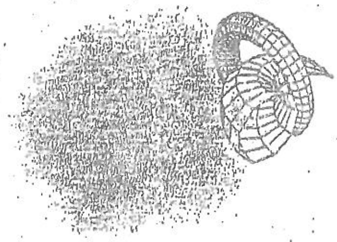


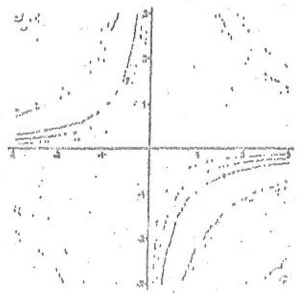
Borough of Manhattan Community College

Mathematics Department





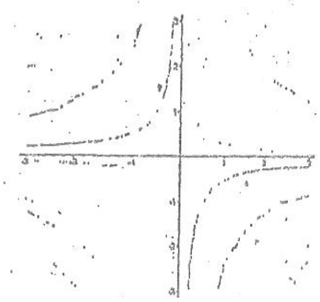


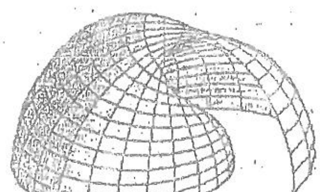


Maple Handbook

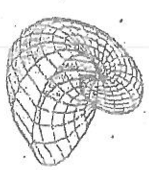
By

Mark Jagai





A study guide for Calculus students using Maple



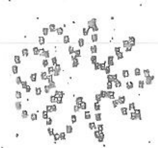


Table of Contents

Introduction

This Maple Handbook is a brief reference tool for the Maple language, and is written for all Maple users at Borough of Manhattan Community College, regardless of their fields of interest. The goal is to provide you with examples of Maple syntax and a more detailed description of some of the Maple commands used most frequently in the Calculus Labs at BMCC. Most of the built-in mathematical, graphical, and system-based commands available in Maple 9.5 Release are listed.

Please note that in most cases, this Maple Handbook does not teach the mathematics behind Maple commands. If you do not know the meaning of such concepts as derivative, definite integral, or functions, do not expect to learn them here. This tutorial was developed to provide a brief and efficient introduction to Maple for students about to enter a Calculus course. Therefore, the tutorial only assumes familiarity with mathematics at the precalculus level. It is highly recommended that you also read a more thorough tutorial such as Introduction to Maple by Andr'e Heck or one of the manuals shipped with Maple Release 9.5

Maple's On-line Help System (www.mapleapps.com)

All versions of Maple come with an exhaustive on-line help facility containing hundreds of pages of detailed descriptions. These help pages have been updated during the many years of Maple development and contain much information that is valuable and much that is esoteric. On many platforms, there are on-line topic browsers that allow you to navigate easily between various help pages.

The on-line help pages can be used to broaden your knowledge at your leisure. Using the on-line help facility is one of the first lessons you should learn about Maple, so that for topics of particular interest you can seek out more detailed information than what is provided in this Maple Handbook.

Written by: Mark Jagai
Edited by: Dr. Glenn Miller, BMCC
Revised by: Alicia N. Lawson, BMCC, 2004

Special Thanks To:
Professor Jack Drucker
For special input and guidance in creating this Handbook

What is Maple?

Maple 9.5 is an advanced computer software tool for doing complicated mathematics quickly and precisely on a computer. It is designed to aid scientists, engineers, students and mathematicians in performing difficult or laborious mathematical calculations. Maple 9.5 is an interactive program that permits the user to enter commands at the prompt, press the return key, and read or use the output to aid in further calculations. You can use the following arithmetical operations: +, -, *, /, ^ , which are addition, subtraction, multiplication, division, and raising to a power, respectively.

Section 1: Numerical Calculations

If you ever need additional information about any Maple command, you can always ask for help by using a command line. This can be done by typing in a question mark followed by the Maple command and then pressing [Enter]. An example of this is shown below:
Note: The help menu will open in another window.

```
> ?addition
```

When using Maple you type in commands at the keyboard and then press [Enter]. Maple 9.5 provides a result. The Maple command line for help is an exception to the rule, as shown above. Every Maple 9.5 command, must be punctuated with either a semicolon ";" or a colon ":". For example, if you wish to multiply two numbers like "247" and "3756", the command would be to type in 247*3756; at the Maple 9.5 prompt and then press [Enter].

Multiplication: Try this now:

```
> 247*3756;
                                927732
```

By pressing [Enter], you also move the cursor to the next command line (i.e. 210 + 375;) in the worksheet.
Addition: Try this now:

```
> 210+375;
                                585
```

What happens if you use a colon?

```
> 247*3756:
```

Notice that no results will be printed, on the other hand the computation is made as we will see later. What happens if you omit the semicolon or colon? The following segment shows that the semicolon is omitted in the first command, and a calculation is made which results in a syntax warning, telling you that the semicolon is missing.

```
> 247*3756
Warning, inserted missing semicolon at end of statement, 247*3756;
                                927732
```

You are expected to complete the statement with the missing semicolon. For example, the above sequence should have been as follows:

Example of multiplication:

```
> 247*3756;
```

$$927732$$

We will now show some examples that illustrate how to perform the elementary arithmetical operations.

You can add two numbers:

Example of addition:

```
> 253+7775;
```

$$8028$$

You can add fractions:

```
> 25/27+3/51;
```

$$\frac{452}{459}$$

The Maple operator of calling the previous output is the % sign. If you wish to multiply the above answer by a constant you may enter the following:

```
> 23*%;
```

$$\frac{10396}{459}$$

If you wish to convert your answer to a decimal you could use the *evalf* command which will be shown in "The Numerical Approximations command" section, or you could type the number using a decimal point, as shown below.

Recall from above that the Maple operator of calling the previous output is the % sign, since you would like to call two previous outputs, you must use double %% sign.

```
> 23.*%%;
```

$$22.64923747$$

One can raise a number to a power:

```
> 3^7;
```

$$2187$$

A feature of most computer algebra systems is that they use exact arithmetic. For example, if you divide two integers Maple 9.5 returns an exact answer.

```
> 3235/7478;
```

$$\frac{3235}{7478}$$

The Numerical Approximations command

There is a built-in Maple 9.5 function *evalf* - evaluate using floating point arithmetic, that will yield the decimal to any degree of accuracy.

Recall: By entering the % sign, you are referring to the previous output.

```
> evalf(%);
```

$$0.4326023001$$

The default number of digits used in floating point output is 10, but if you wish to have any other number of digits then you can specify them using *evalf*. The following is the 30-digit floating-point approximation of the fraction 3235/7478.

By entering % %, you are referring to the output from two commands above.

```
> evalf(%%,30);
```

$$0.432602300080235357047338860658$$

You can assign a value or a function to a variable with the colon-equal symbol " = ".

```
> A:=5;
```

$$A := 5$$

```
> A;
```

$$5$$

This means that the variable "A" has been assigned the value 5 and it will have this value through the remainder of the session unless you assign it another value.

It is recommended to create an assignment statement or define a function before it is being used in any Maple command. This can be seen in the proceeding section.

Section 2: Algebraic Calculations

In this section you will learn some very important commands that will be used throughout your calculus sequence. One of the most important and confusing concepts to students is the way we assign a name to a function and the other is the procedure to enter a function on a Maple work sheet as a function. We will start with the basics which is how to enter an algebraic expression and substitute values in for the variables. Then you will learn the commands that will allow you to *expand*, *factor* and *simplify* expressions.

```
> B:=x^3+7*x-2;
```

$$B := x^3 + 7x - 2$$

```
> B;
```

$$x^3 + 7x - 2$$

If we wish to use the above function, we can now refer to it as "B".

```
> 4*B+12;
```

$$4x^3 + 28x + 4$$

The following statement is the way to "unassign" the variable.
B is enclosed within two single quotes, which is the way to "unassign" the variable.

```
> B:='B';
```

$$B := B$$

```
> f:=x^2;
```

$$f := x^2$$

The last command assigns x^2 to the letter f, and you can check that with the following command.

```
> f;
```

$$x^2$$

There is a Maple 9.5 procedure called *subs* which allows you to evaluate an expression. The format is *subs(variable=numerical value, expression involving variable).*

```
> subs(x=5,f);
```

$$25$$

A word of warning here. Many beginners want to use standard functional notation for a Maple 9.5 assignment statement, such as f(5). However, Maple interprets this as multiplication. Recall from above that f is assigned the variable x^2.

```
> f(x);
```

$$x(x)^2$$

```
> f(5);
```

$$x(5)^2$$

If you would like to use standard functional notation then you can do so using the symbol "->", made by typing the "minus sign" followed by the "greater than" sign. NOTE: There is a major difference between assigning an expression of a variable and entering as a function.

```
> f:=x->x^2;
```

$$f := x \rightarrow x^2$$

```
> f(x);
```

$$x^2$$

Now we have the above result that is a function of f(x). This is not the same as if you try to type f, as shown below.

```
> f;
```

$$f$$

```
> f(5);
```

$$25$$

If you already have *g* defined as an expression and want to convert it to a function f(x), you can use the Maple 9.5 command *unapply(expr,x)*. For example:

```
> g:=x^3;
```

$$g := x^3$$

```
> f:=unapply(g,x);
```

$$f := x \rightarrow x^3$$

```
> f(x);
```

$$x^3$$

```
> g:=x^3;
```

$$g := x^3$$

The expression g does not evaluate since it is not entered as a function.

```
g(5);
```

$$x(5)^3$$

One of the great benefits of a computer algebra system such as Maple 9.5 is that it allows you to manipulate algebraic expressions in much the same way that a calculator permits you to manipulate numbers. Some of the algebra commands used in Maple 9.5 are listed below:

simplify -- simplifies an algebraic expression

expand -- expands an expression

factor -- factors an expression

solve -- solve a system of equations for a set of unknowns

It is very important to insert the parenthesis in the correct order, since incorrect order may change the function. However, Maple also simplifies the function as it produces the output.

```
> simplify((1+x)/x+(1-x)/x);
```

$$\frac{2}{x}$$

```
> expand((x^2-4)*(x+1)*(x-2)*(x^2+x+1));
```

$$-6x^4 - 3x^3 + 6x^2 + x^6 + 12x + 8$$

```
> factor(%);
```

$$(x+2)(1+x)(x^2+x+1)(x-2)^2$$

The command *solve(eqn,var)* finds exact solutions to polynomial equations.

Example:

The exact solution of the polynomial equation $3x^3 - 4x^2 - 43x + 84 = 0$ can be found by entering:

```
> solve(3*x^3-4*x^2-43*x+84=0,x);
```

$$3, -4, \frac{7}{3}$$

As you can see, the second argument tells Maple that *x* is the unknown variable that we are solving for.

The *solve* command can also be used to find the exact solutions for non-polynomial equations.

Example: Solve the equation $5\,e^{\left(\frac{x}{4}\right)} = 43$:

Some expressions may be written differently on Maple worksheet. One such expression is *e*, in Maple *e* is written as *exp(1)*. The *(1)* represents the implied exponent 1 when we write *e*. Therefore the above expresion must be entered as *exp(x/4)*, since the exponent is not 1.

```
> solve(5*exp(x/4)=43);
```

$$4\ln\left(\frac{43}{5}\right)$$

Notice that Maple did not give an approximate solution as the solution above.

Maple's *fsolve* command can be used to find approximate solutions for any equation. In the following polynomial equations, *fsolve* produces a complete list of all the real solutions.

```
> eqn:=x^4-x^3-17*x^2-6*x+2=0;
```

$$eqn := x^4 - x^3 - 17x^2 - 6x + 2 = 0$$

```
> fsolve(eqn,x);
```

$$-3.414213562, -0.5857864376, 0.2087121525, 4.791287847$$

The four solutions listed above provide us with the complete list of solutions to the equation.

Below is an example showing the command for solving system of equations using *solve* and *fsolve*.

```
> sol:=solve({2*x-5*y=12,12*x+4*y=17},{x,y});
```

$$sol := \left\{ y = \frac{-55}{34}, x = \frac{133}{68} \right\}$$

```
> sol2:=fsolve({2*x-5*y=12,12*x+4*y=17},{x,y});
```

$$sol2 := \{y = -1.617647059, x = 1.955882353\}$$

Observe that we used curly brackets "{" and "}" in the preceding Maple 9.5 input. Maple 9.5 uses this notation for a set. Observe that the Maple 9.5 output for the expression "sol" is in the form of a set. One thing to notice about sets is that they do not distinguish as to order. For example, Maple 9.5 might instead have produced an equivalent output with a different order. Maple 9.5 regards a set as a kind of array and you can pick out of its elements. For example, the last Maple 9.5 output indicated above is a set consisting of two elements: the first is the equation y=-55/34, and the second element is the equation x=133/68. The way to select the first element is by entering *sol[1]*; and pressing *[Return]*.

```
> sol[1];
```

$$y = \frac{-55}{34}$$

The second element is obtained by entering *sol[2]*; and pressing *[Return]*.

```
> sol[2];
```

$$x = \frac{133}{68}$$

You may use *subs* to check your answer.

```
> subs(sol,{2*x-5*y=12,12*x+4*y=17});
```

$$\{12 = 12, 17 = 17\}$$

Observe that we have employed three different kinds of grouping symbols: "( )", "{ }" and "[ ]". They are used for different purposes and Maple 9.5 requires that you use them correctly. The standard parentheses "( )" are used in functions as in *factor(x^2-1)*, or *sin(Pi)*. The curly brackets "{ }" are used to group a set of things together as in "{x, y}". The square brackets are used to pick a coordinate out of a group as in "*sol[2]*". There will be other examples to illustrate the use of these grouping symbols in the future. Finally, you can only use parentheses "( )" when grouping symbols in mathematical expressions, for example, a command like {2*[5+3*(7+2)]*(12+3)}+2; may make sense mathematically, but is not interpreted as you might expect by Maple.

```
> {2*[5+3*(7+2)]*(12+3)}+2;
```

$$\{[960]\} + 2$$

You should type in the following:

```
> (2*(5+3*(7+2))*(12+3))+2;
```

$$962$$

Complete the square

completesquare(expr) -- completes the square of polynomials of degree 2 in x by re-writing the polynomial as a perfect square plus a remainder

completesquare(expr,x) -- completes square with respect to x in expression expr

completesquare(expr,y) -- completes square with respect to y in expression expr

completesquare(expr,{x,y}) -- completes square with respect to x and y in expression expr

A special package called *student* must be used before some commands can be excuted. The *student* package is a collection of subpackages designed to assist with the teaching and learning of standard undergraduate mathematics. There are many routines for displaying functions, computations, and theorems in various ways. There is also support for stepping through important computations.

In addition the package has a collection of commands designed to carry out step-by-step solutions to problems. To use these commands, you need to execute the following line which loads the *student* package. Recall, the colon at the end of the statement allows this line to be executed without displaying any distracting output. To see the contents of the *student* library, you can change the colon to a semicolon.

Example:

```
> with(student):
> completesquare(9*x^2 + 24*x + 16);
```

$$9\left(x+\frac{4}{3}\right)^2$$

Example 2: Find the vertex of the parabola given by $9x^2+4y^2+36x-24y+36=0$.

It is recommended that you assign the above function to a letter as shown below

```
> f:=9*x^2+4*y^2+36*x-24*y+36=0;
```

$$f := 9x^2+4y^2+36x-24y+36=0$$

At this point Maple can complete the square of the above output in x.

```
> step1:=completesquare(f,{x,y});
```

$$step1 := 4(y-3)^2-36+9(x+2)^2=0$$

We can add 36 to both sides then divide through by 36 to achieve a "1" on the right side:

```
> step2:=lhs(step1)+36 = rhs(step1)+36;
```

$$step2 := 4(y-3)^2+9(x+2)^2=36$$

```
> lhs(step2)/36=rhs(step2)/36;
```

$$\frac{1}{9}(y-3)^2+\frac{1}{4}(x+2)^2=1$$

The vertex (h,k) is at the point (-2,3).

Section 3: Graphing

During your course work for Calculus I, some of the basic functions are as follows. Please be aware of the shape of the graph of these functions and the details of these functions:

```
> a:=x;
```

$$a := x$$

```
> b:=x^2;
```

$$b := x^2$$

```
> c:=x^3;
```

$$c := x^3$$

```
> d:=sqrt(x);
```

$$d := \sqrt{x}$$

```
> f:=abs(x);
```

$$f := |x|$$

```
> g:=a^x;
```

$$g := x^x$$

```
> h:=log10(x);
```

$$h := \frac{\ln(x)}{\ln(10)}$$

```
> k:=exp(x);
```

$$k := e^x$$

```
> l:=sin(x);
```

$$l := \sin(x)$$

```
> m:=cos(x);
```

$$m := \cos(x)$$

```
> n:=tan(x);
```

$$n := \tan(x)$$

Assuming you know the basic shape of the graphs of the above functions, let's learn the basic steps in plotting a function.

A basic call to the plot function is *plot(f(x), x=a..b)*, where *f* is a real function in *x* and *a..b* specifies the horizontal real range on which *f* is plotted. A more typical call to plot a function is *plot(f(x), x=a..b,y=a..b,thickness= n,color=black)*. The function in *x*, and *y=a..b*, specifies the vertical real range on which *f* is plotted. Thickness is an option that specifies the thickness of lines in the plot. The thickness must be a non-negative integer. The default thickness is 0. To specify color for plotting 2-D plots, use the predefined color names which are predefined in Maple:
aquamarine, black, blue, navy, coral, cyan, brown, gold, green, gray, grey, khaki, magenta, maroon, orange, pink plum, red, sienna, tan, turquoise, violet, wheat, white, yellow

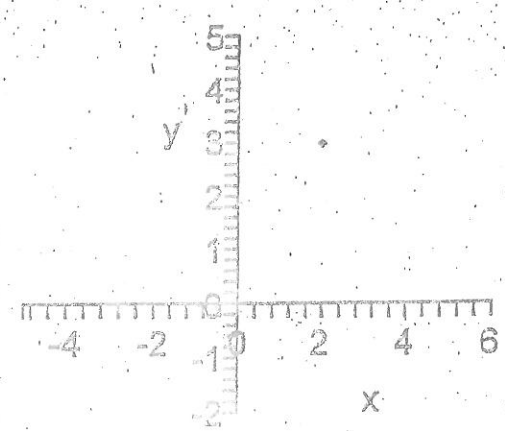
Multiple functions can be plotted by placing a set or list of functions in the first argument. Options such as color, thickness and style can be specified for each function by placing a list in the appropriate option argument.
(See the follwoing plot commands listed below)

In this section you will learn how to plot points and how to plot the graph of a function. We will also look at plotting multiple graphs on the same x-y rectangular coordinate system, and many other commands that will be useful while plotting a graph.

Plotting Points

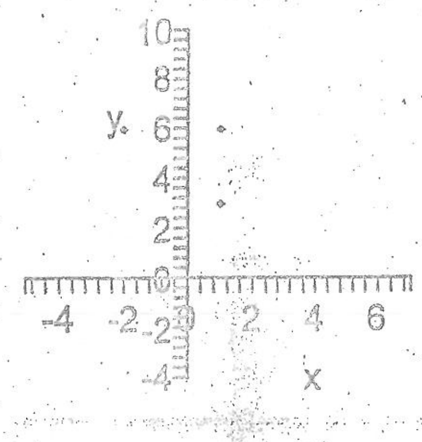
If points or graphs are being plotted, we must enter the domain which is the x-axis boundary, and, if you wish you can also enter the range. It is helpful when plotting points or graphs to include the style of the point as shown on the graph as shown below. The style, will indicate what design you wish to see on the graph. Maple is limited on the different types of style. The interpolation style must be one of LINE, POINT, PATCH, or PATCHNOGRID. The default is LINE. POINT, style plots points only, LINE interpolates between the points, PATCH uses the patch style for plots containing polygons, and PATCHNOGRID is the PATCH style without the grid lines.

```
> plot({[2,3]},x=-5..6,y=-2..5,style=point);
```



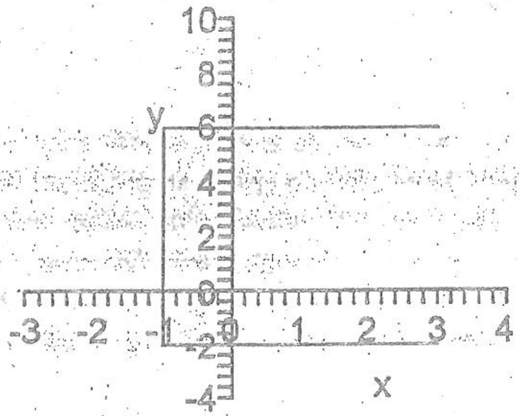
Plotting multiple points:

```
> plot({[1,3],[-2,6],[1,6]},x=-5..7,y=-4..10,style=point);
```



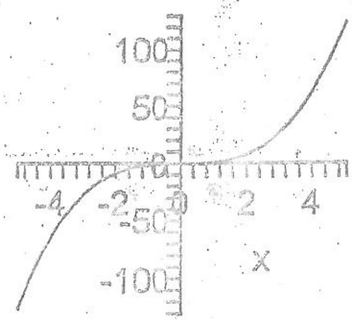
Changing the style to "line" connects the points in the order as listed.

```
> plot({[3,6],[-1,6],[-1,1],[-1,-2],[3,-2]},x=-3..4,y=-4..10,style=line);
```



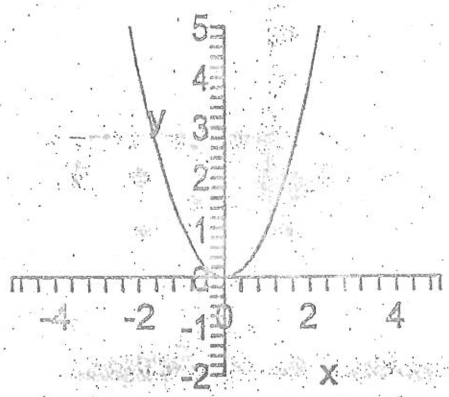
Plotting Graphs:
Recall the command to plot a graph or a point is *plot*. Below we are going to plot the function of $y = x^3$

```
> plot(x^3,x=-5..5);
```



As we can see from the above diagram the function is defined on the x-axis from x = -5 to x = 5. If you wish you can also enter the values for the y-axis as shown below.

```
> plot(x^2,x=-5..5,y=-2..5);
```



In this case both the x-axis and the y-axis has the values that were entered by the user. As we continue we will further explore function plotting capabilities.

On some occasions we may prefer to plot multiple functions on the same axis, this is done using the two functions below.

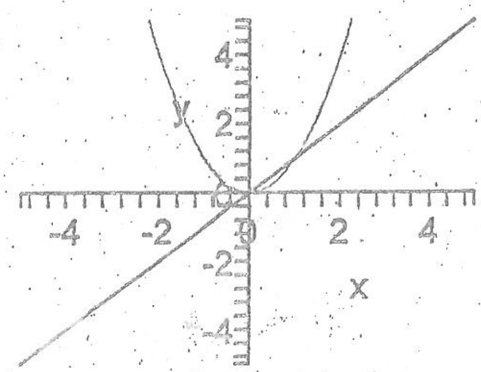
```
> a:=x;
```

$$a := x$$

```
> b:=x^2;
```

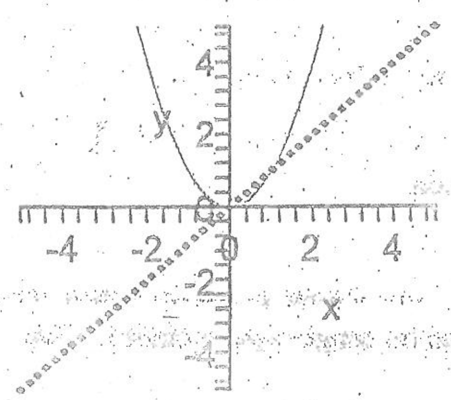
$$b := x^2$$

```
> plot({a,b},x=-5..5,y=-5..5);
```



If you wish to display the graphs using different styles, we can use two of the various style options. Recall, the interpolation style must be one of LINE, POINT, PATCH, or PATCHNOGRID. In the proceeding example we choose POINT and LINE. As shown, the first style will represent the first function and the second will represent the second function.

```
> plot([a,b],x=-5..5,y=-5..5,style=[point,line]);
```



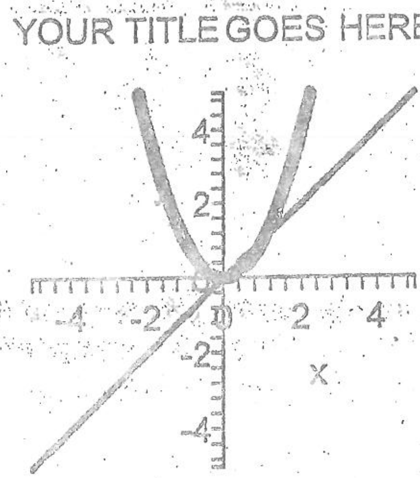
Combining Graphs of function and Points

Recall of special packages:

A special plotting package called *plots* contains many additional graphing features. To use these commands, you need to execute the following line which loads the plot package. Recall, the colon at the end of the statement allows this line to be executed without displaying any distracting output. To see the contents of the plot library, you can change the colon to a semicolon.

As you become more familiar with Maple, you will learn about the library, defining a plot statement and placing a title on the graph. We can start by opening the library *with(plots)* entering the *plot* statement and ending it with a colon, this is because the new command called *display* puts two separately created plots into one graph. Let's try this using the same functions as in the previous exercise:

```
> with(plots):
Warning, the name changecoords has been redefined
> G:=plot(x,x=-5..5,y=-5..5,thickness=2):
> H:=plot(x^2,x=-5..5,y=-5..5,thickness=4):
> display({G,H},scaling=constrained,title="YOUR TITLE GOES HERE");
```



Lab Activity #1:

a) Create an assignment statement for each of the above functions.

b) Use the *subs* command and find the value for *f* and *g* when x = 4.

c) Define *f(x)* and *g(x)* above as a function.

d) Find *f(4)* and *g(4)*

e) Use the plot command and *plot* what you entered in part (c) above on the same rectangular coordinate system. You may choose the domain and the range of your choice. (*Hint: remember you defined f and g as functions in part (c)).*

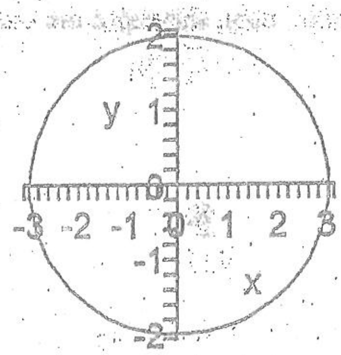
Graphing of Implicit Functions

The command *implicitplot(equation,x=a..b,y=c..d)* graphs a relation with variables x and y on a horizontal real range of [a, b] and vertical real range of [c, d].

Example: The graph of the curve $4x^2 + 9y^2 = 36$ is as follows using the *implicitplot* command:

```
> with(plots):
```

```
> implicitplot(4*x^2+9*y^2=36,x=-5..5,y=-5..5);
```



Graphing of Logarithm and Exponential Functions

ln(x)

log(x)

log[b](x)

log10(x)

- The log function is the general logarithm. For x > 0 and b > 0 we have log[b](x) = y <==> x = b^y. *Log* is extended to general complex b and x by log[b](x) = ln(x)/ln(b).

- The default value of the base b is exp(1).

- *log10(x) = log[10](x).*

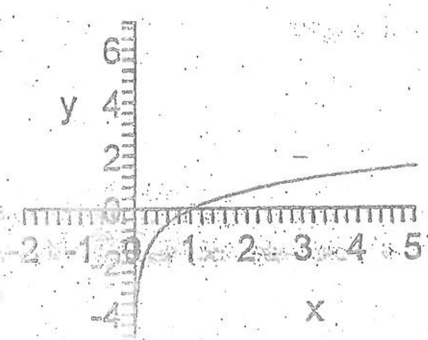
Examples:

```
> ln(1);
```

0

This can also be seen graphically as follows:

```
> plot(ln(x),x=-2..5,y=-5...7);
```



As shown above as the function pass through the x-axis at $x = 1$, the y value is 0, therefore ln(1) = 0.

Recall from the description that *log[b](x)* = ln(x)/ln(b), therefore the output from Maple may not be identical to what was entered.

```
> log[10](x);
```

$$\frac{\ln(x)}{\ln(10)}$$

Another way to enter the above command is without the brackets.

```
> log10(x);
```

$$\frac{\ln(x)}{\ln(10)}$$

```
> log10(10000);
```

$$4$$

If you wish to see the actual decimal number for the above then you may use the *simplify* command or the *evalf* command.

Recall that the % sign assigns the previous Maple output.

```
> evalf(%);
```

$$4.$$

In some cases if you are using different bases other than base 10 then you have to use the bracket as shown below.

```
> log[2](x);
```

$$\frac{\ln(x)}{\ln(2)}$$

```
> log[2](16);
```

$$4$$

Recall: the % sign assigns the previous Maple output.

```
> simplify(%);
```

$$4$$

Maple is capable of computing the value of any base within a fraction of a second. For practice you can try entering any logarithm function with different bases and see what the output would look like.

The Exponential Function

The exponential function, *exp(x)*, calculates the value of *e* to the power of *x*, where *e* is the base of the natural logarithm, 2.718281828... .

e is no longer reserved in Maple, *exp(1)* is used instead.

To obtain the value of *e*, you must enter the following:

```
> exp(1);
```

$$e$$

```
> evalf(%);
```

$$2.718281828$$

It is a common mistake to type *exp(x)* to obtain the number 2.718281828

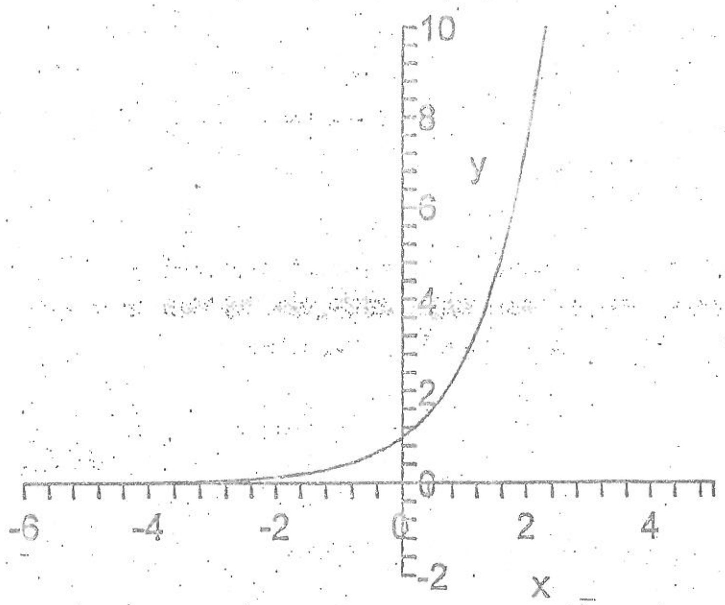
If *exp(x)* is entered it means that you are entering e^x

```
> exp(x);
```

$$e^x$$

To plot the above function you would enter the following:

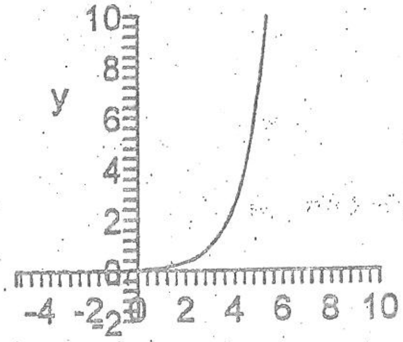
```
> plot(exp(x),x=-6..5,y=-2..10);
```



If you would like to shift the above function horizontally by n units we would enter *exp(x + n)* or *exp(x - n)*. This can be seen below.

Example: The function e^x shifted three units to the right is as follows:

```
> plot(exp(x-3),x=-5..10,y=-2..10);
```



Lab Activity #2:

a) Plot the graphs of *f(x)* and *g(x)* on the same x-y coordinate system.

b) Plot the graphs of *h(x)*, *k(x)*, and *n(x)* on the same x-y coordinate system.

c) Plot the graphs of *f(x)* and *h(x)* on the same x-y coordinate system.

d) From the graphs in part c, explain the relationship between *f(x)* and *h(x)*. Hint: (you can look at the inverse of either one of the functions)

Section 4: Functions

Piecewise Functions

Let's look at the piecewise function. The syntax is the same as in a case statement: if cond_1 is true, then f-1, else if cond_2 is true then f_2 and so on. Otherwise gives a default case which corresponds to all conditions being false. The default for f_otherwise is 0.

A condition can be a single equality or inequality, or a boolean combination of inequalities; e.g. x<3, x>0 and x<=Pi. The condition can contain relations with polynomials, abs, signum, or piecewise functions; e.g. x^2 - 4>0 and x>0, abs(x)<4. In all cases, x is assumed to be a real variable.

```
> piecewise(x>0,x);
```

$$\begin{cases} x & 0 < x \\ 0 & otherwise \end{cases}$$

```
> piecewise(x<0,1/x,x=2,2,x>2,1/(x+3));
```

$$\begin{cases} \frac{1}{x} & x < 0 \\ 2 & x = 2 \\ \frac{1}{x+3} & 2 < x \end{cases}$$

If you enter a piecewise as a function it will appear as shown below. It is the same function as above, however, it is entered in Maple as a function.

```
> h:=x->piecewise(x<0,1/x,x=2,2,x>2,1/(x+3));
```

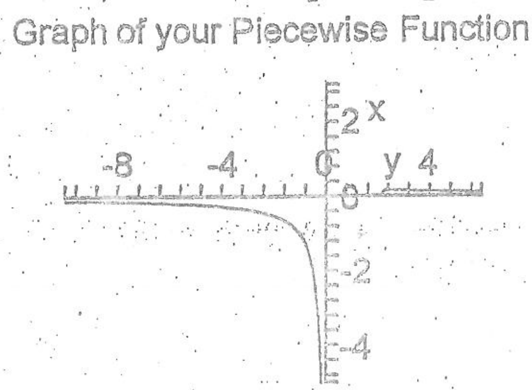
$$h := x \rightarrow piecewise\left(x < 0, \frac{1}{x}, x = 2, 2, 2 < x, \frac{1}{x+3}\right)$$

```
> h(4);
```

$$\frac{1}{7}$$

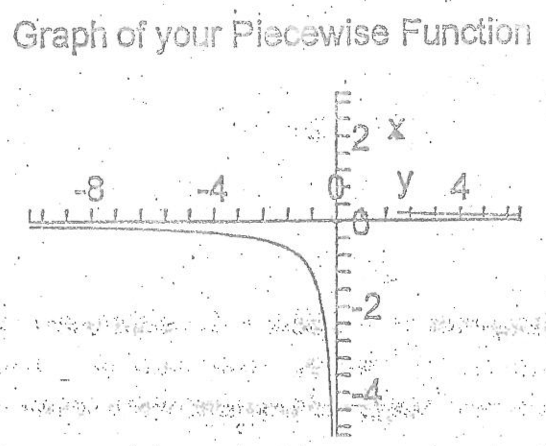
The graph below is incorrect. Can you find the errors?

```
> plot(h(x),x=-10..6,y=-5..3,title="Graph of your Piecewise Function");
```



Use *discont = true* parameter to obtain the correct graph.

```
> plot(h(x),x=-10..6,y=-5..3,discont=true,title="Graph of your Piecewise Function");
```



The *discont=true* command will show most of the points of discontinuity. However, the function may still need some work to be correct. As shown above, the point x = 2, should have been a solid dot. This is one of the bugs that Maple may fix in the future.

Rational Functions

In this section you will learn how to analyze rational functions, e.g., functions that can be written as the quotient of two polynomials. Both local and global behavior of these functions will be examined by studying zeros, poles and slant asymptotes. Representing rational functions in terms of partial fraction is introduced.

Definition: A rational Function is a function which is the quotient of polynomial functions, e.g., rational functions are functions of the form:

$f(x)=p(x)/q(x)$, where $p(x)$ and $q(x)$ are polynomial functions.

In the particular case that $q(x)$ is a constant function, f reduces to a polynomial function.

```
> f:=x->x^2*(x-2)/((x-1)^2*(x+1));
```

$$f := x \to \frac{x^2\,(x-2)}{(x-1)^2\,(x+1)}$$

Maple 9.5 enables you to select the numerator or denominator of $f(x)$ with the commands *numer* and *denom*, respectively.

```
> numer(f(x));
```

$$x^2\,(-2+x)$$

```
> denom(f(x));
```

$$(x-1)^2\,(x+1)$$

Note: One can thus find the zeros (i.e., points when the numerator of $f(x)$ is zero) and the poles (i.e., points when the denominator of $f(x)$ is zero) of $f(x)$ with the commands:

```
> solve(numer(f(x))=0,x);
```

$$2, 0, 0$$

```
> solve(denom(f(x))=0,x);
```

$$-1, 1, 1$$

Note: The poles give the vertical asymptotes $x = 1$, and $x = -1$, points where the function f is not defined and for which, in the case of rational functions, $|f(x)|$ becomes unbounded as x approaches one of the points. For purposes of illustration, we present the next Maple 9.5 segment which shows that as x approaches the number 1, then $f(x)$ decreases without bound.

Let us now explore a little further:

Lab Activity # 3:

a) Separate the numerator of the rational function and assign it to the variable B. Hint: use the *numer* command.

b) Separate the denominator of the rational function and assign it to the variable C. Hint: use the *denom* command.

c) Find the two binomial factors of the numerator. Hint: use the *factor* command

d) Find the zeros of the above function. Hint: you can solve for x from the results in part B.

e) Use the *simplify* command to simplify the function *f* and explain your results.

Section 5: LIMITS

limit(f(x),x=a) --limit of f(x) as x approaches a

limit(f(x),x=a,left) -- limit of f(x) as x approaches a from the left

limit(f(x),x=a,right) -- limit of f(x) as x approaches a from the right

Limit(f(x),x=a) -- inert form of limit, does not compute the numerical output

Note: Since the *Limit* command line does not evaluate or check the existence of the limit of the expression, it can lead to incorrect transformations. Therefore, the use of *limit* is more reliable.

The limit function attempts to compute the limiting value of "x" as it approaches "a".

Now let's try this, if we would like to use the limit notation, then using capital "L" for Limit (inert form)

```
> Limit(x+2,x=5);
```

$$\lim_{x \to 5} (x+2)$$

To evaluate this and obtain an actual numerical output, use lower case *l*

```
> limit(x+2,x=5);
```

$$7$$

Another way of evaluating limits is by assigning the function:

```
> f:=x+2;
```

$$f := x + 2$$

```
> limit(f,x=5);
```

$$7$$

Look at a rational function we solved earlier and try to find the limit:

```
> f:=x->x^2*(x-2)/((x-1)^2*(x+1));
```

$$f := x \to \frac{x^2 (x-2)}{(x-1)^2 (x+1)}$$

Maple 9.5 enables you to select the numerator or denominator of *f(x)* with the commands *numer* and *denom*, respectively.

```
> numer(f(x));
```

$$x^2(-2+x)$$

```
> denom(f(x));
```

$$(x-1)^2(x+1)$$

```
> solve(numer(f(x))=0,x);
```

$$2, 0, 0$$

```
> solve(denom(f(x))=0,x);
```

$$-1, 1, 1$$

Note: The poles give the vertical asymptotes $x=1$, and $x=-1$, points where the function $f(x)$ is not defined and, in the case of rational functions, $|f(x)|$ becomes unbounded as x approaches one of the points.

Limits using Maple's spreadsheet

When trying to obtain the a visual display of information about limits in a tabular form, you can use a Maple spreadsheet.

1. From the **Insert** menu, select **Spreadsheet**. The **Spreadsheet Title** dialog opens (optional).

2. Enter a title for the spreadsheet.

3. Click **OK**. The spreadsheet opens in the worksheet.

To enter data in cells:

There are several methods for entering text and equations. The most straightforward method is by entering information into cells.

1. Click a cell to select it.

2. Enter the new expression.

3. Press the **Enter** key.

When you change the contents of a cell in a spreadsheet, the formula in that cell, or in any cell referencing the changed cell, the content of the cell is not automatically recalculated. Instead, Maple draws cross hatches over the cell identifying it as a **Stale** cell. Evaluate the cell to refresh its value.

1. Select the cell or group of cells containing cross hatches.

2. From the **Spreadsheet** menu, select **Evaluate Selection**.

To resize a spreadsheet:

A spreadsheet can be resized to any dimension that fits in your current worksheet.

1. Click the outside border of the Spreadsheet. A double-lined box with small black boxes appears around the perimeter.

2. Click one of the black boxes on the perimeter and drag to the size desired.

3. Release the mouse button.

To hide or show the column and row headings in a spreadsheet:

From the **Spreadsheet** menu, select the **Show Headers** check box to activate or deactivate the feature. A check mark next to **Show Headers** indicates the borders are visible.

Example of a spreadsheet:

```
> f:=x->x^2*(x-2)/((x-1)^2*(x+1));
```

$$f := x \to \frac{x^2\,(x-2)}{(x-1)^2\,(x+1)}$$

After inserting the spreadsheet, it is best to restart the Maple program by clicking on the Restart Maple server icon on the menu bar.

Doing this will allow the user to write f(x) without having the actual function in the cell. One would have to re-execute the definition of the desired function f(x) as above.

To compute the value of the function for the first x-value, you can enter f(0.9) and press the enter key. Since the spreadsheet is active, the value would be calculated automatically. You can obtain the other values of f(x) by doing a similar procedure, with the different x-values.

Limit as x approaches 1								
	A	B	C	D	E	F	G	H
1	x	0.90000	0.99000	0.99900	1	1.0001	1.0010	1.0100
2	$f(x)$	-46.895	$f(0.99000)$	$-4.997\ 10^5$	Error	$-5.002\ 10^5$	-5024.4	-5024.4

Note: When f(1) is enter in the cell above, you have an Error result. This is because if the value of x = 1 is substituted into the function f(x), the denominator will be zero, which is an illegal operation.

The next Maple 9.5 command shows that as x approaches the number 1, then f(x) decreases without bound.

```
> limit(f(x),x=1);
```

$$-\infty$$

Note: On the other hand, as x approaches the number -1, the value of $f(x)$ actually either increases without bound or decreases without bound depending on weather x is to the left of -1 or to the right of -1, respectively.

```
> limit(f(x),x=-1);
```

undefined

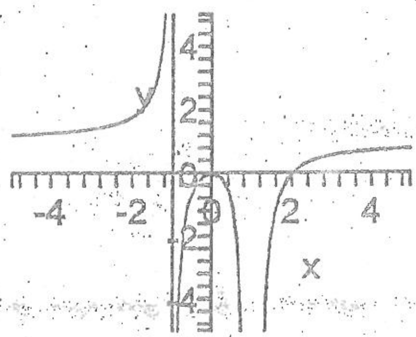
```
> limit(f(x),x=-1,left);
```

∞

```
> limit(f(x),x=-1,right);
```

$-\infty$

```
> plot(f(x),x=-5..5,y=-5..5);
```



If the function *f(x)* approaches a constant value as |x| becomes a large positive number, then we say *f(x)* has a horizontal asymptote. The horizontal asymptotes, if they exist, can be obtained as follows: (the answers must be the same before we can say there is a horizontal asymptote.

```
> limit(f(x),x=infinity);
```

1

```
> limit(f(x),x=-infinity);
```

1

Section 6: Differentiation or Partial Differentiation

You can enter the differentiation command as follows:

diff(a, x1, x2, ..., xn)

OR

Diff(a, x1, x2, ..., xn)

OR

diff(a, [x1, x2, ..., xn])

OR

Diff(a, [x1, x2, ..., xn])

- *diff(f(x),x)*, computes the derivative of the function *f(x)* with respect to *x*.

- The sequence operator $ is useful for forming higher-order derivatives. *diff(f(x),x$4)*, for example, is equivalent to *diff(f(x),x,x,x,x) OR (D@@n)(f)(x)*

- The capitalized function name *Diff* is the inert *diff* function, which simply returns unevaluated. The pretty printer understands *Diff* to be equivalent to *diff* for printing purposes but formats the derivative in black to visually distinguish the inert case.

Before looking at the derivative command, we should explore its definition as shown below:

```
> Limit(((f(x+h)-f(x))/h),h=0);
```

$$\lim_{h \to 0} \left(\frac{\frac{(x+h)^2 (x+h-2)}{(x+h-1)^2 (x+h+1)} - \frac{x^2 (-2+x)}{(x-1)^2 (x+1)}}{h} \right)$$

We can assume h = $\Delta(x)$

If we now try to use this definition to evaluate a particular derivative, we will proceed as follows:

```
> f:=x->(x+2)^2;
```

$$f := x \to (x+2)^2$$

```
> Limit(((f(x+h)-f(x))/(h)),h=0);
```

$$\lim_{h \to 0} \left(\frac{(x+h+2)^2 - (x+2)^2}{h} \right)$$

To obtain a numerical output, remember to use a lower case "l" when typing the word limit.

```
> limit(((f(x+h)-f(x))/(h)),h=0);
```

$$2x+4$$

To verify the above definition, use the *diff* command. Using a upper case "D" will show the function, but will not evaluate the derivative.

Let's try the above example by first creating an assignment statement:

```
> f:=(x+2)^2;
```

$$f := (x+2)^2$$

```
> Diff(f,x);
```

$$\frac{d}{dx} ((x+2)^2)$$

```
> diff(f,x);
```

$$2x+4$$

Other examples using the *diff* command.

```
> diff(sin(x),x);
```

$$\cos(x)$$

```
> diff(tan(x),x);
```

$$1 + \tan(x)^2$$

```
> diff(1/x,x);
```

$$-\frac{1}{x^2}$$

You can find the first, second, third or any order derivative using Maple.

Example: Find the velocity and the acceleration of a free-falling object whose position is s(t) = $-16\,t^2 + 100$. Hint: we will have to find the second and third derivatives.

```
> s:=t->-16*t^2+100;
```

$$s := t \rightarrow -16\,t^2 + 100$$

The velocity (first derivative) can be found as follows, since we have defined the function above:

```
> diff(s(t),t);
```

$$-32\,t$$

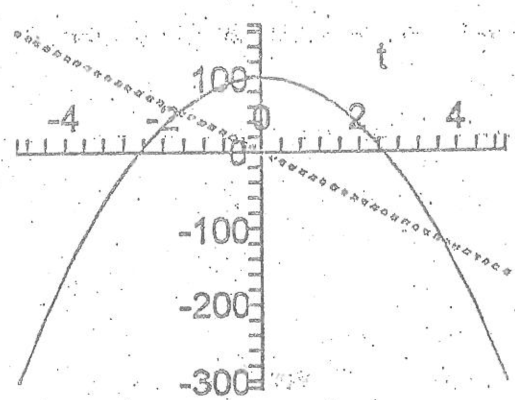
The acceleration (second derivative) can be found as follows:

```
> diff(s(t),t$2);
```

$$-32$$

The graph of s(t) and s '(t) is as follows::

```
> plot({s(t),D(s)(t)},t=-5..5,style=[line,point]);
```



As shown above the first style, line, represents the first function, s(t) and the second style will represent the second function s '(t).

Therefore as stated above, the sequence operator *$* is useful for forming higher-order derivatives; *diff(f(x),x$4)*, for example, is equivalent to *diff(f(x),x,x,x,x)* or *diff(diff(diff(diff(f,x),x),x),x)* which is the fourth derivative.

Implicit Differentiation

The call *implicitdiff(f,y,z)* computes *dy/dx*, the partial derivative of the function *y* with respect to *x*. The input *f* defines y as a function of *x* implicitly. It must be an equation in *x* and *y* or an algebraic expression which is understood to be set equal to zero. For example, the call *implicitdiff(x^2*y+y^2=1,y,x)*; computes the derivative of *y* with respect to *x*. Here, *y* is implicitly a function of *x*. The result returned is -2*x*y/(x^2+2*y).

```
> f:=x^2+4*y^2=4;
```

$$f := x^2 + 4\,y^2 = 4$$

The call *implicitdiff(f,x,y)* computes the *dy/dx* (slope) of the function y with respect to x.

```
> implicitdiff(f,y,x);
```

$$-\frac{x}{4\,y}$$

```
> subs({x=sqrt(2),y=-1/sqrt(2)},%);
```

$$\frac{1}{2}$$

Therefore, at $(\sqrt{2}, -\frac{1}{\sqrt{2}})$ the slope is $\frac{1}{2}$.

Lab Activity #4:

a) Use the definition of the derivative to find the slope of the above function at $x = 2$.

b) Use the differentiation command to find the slope of the above function at $x = 2$.

c) What can you conclude from the above answers?

Lab Activity #5:

a) Enter *f* as a function.

b) Find the slope of the function at x = 3. (Hint: Recall the derivatives can help to find the slope)

c) Find the equation of the tangent line passing through the point (3, 1) and having a slope as found in part *b*.

d) Plot the function and its tangent line on the same coordinate system.

Section 7: Integration by approximation

Leftbox (rectangular boxes with the left tip of the boxes touching the graph). The command *leftbox*, only generates a plot of rectangular boxes used to approximate a definite integral. The height of each rectangle (box) is determined by the value of the function at the left side of each interval. The formula for the corresponding numerical approximation to this integral is generated by the Maple procedure *leftsum*. Leftsum, therefore, does the computation but does not produce the answer. This can be corrected by using the procedure "*value*" or "*evalf*" (recall the procedure *evalf* gives the answer in decimal form).

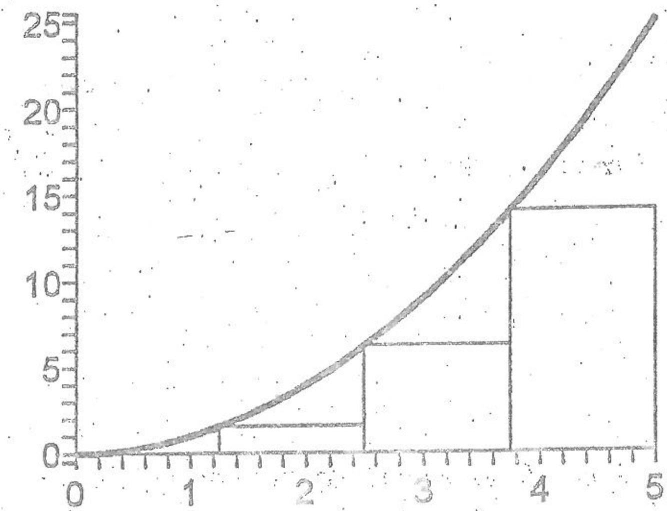
Please note that the library *with(student)* must be open before you can use the command *leftbox*.

The *leftbox(f(x),x=a..b,n)* command graphs f(x) on [a,b], drawing n rectangles under f(x) to approximate the are

The *leftsum(f(x),x=a..b,n)* command find the exact sum of areas of rectangles from the leftbox.

Example: Given the function $y = x^2$ approximate a definite integral from x=0 to x=5.

```
> with(student):
  leftbox(x^2, x=0..5,4,color=RED);
  leftsum(x^2, x=0..5,4);
  value(%);
  evalf(%);
```



$$\frac{5}{4}\sum_{i=0}^{3}\frac{25}{16}i^2$$

$$\frac{875}{32}$$

$$27.34375000$$

Integration by approximation

Rightbox (rectangular boxes with the right tip of the boxes touching the graph)

The command *rightbox*, only generates a plot of rectangular boxes used to approximate a definite integral. The height of each rectangle (box) is determined by the value of the function at the right side of each interval. The formula for the corresponding numerical approximation to this integral is generated by the Maple procedure *rightsum*. Rightsum, therefore, does the computation, but does not produce the answer. This can be corrected by using the procedure "*value*" or "*evalf*" (recall that the procedure *evalf* gives the answer in decimal form).

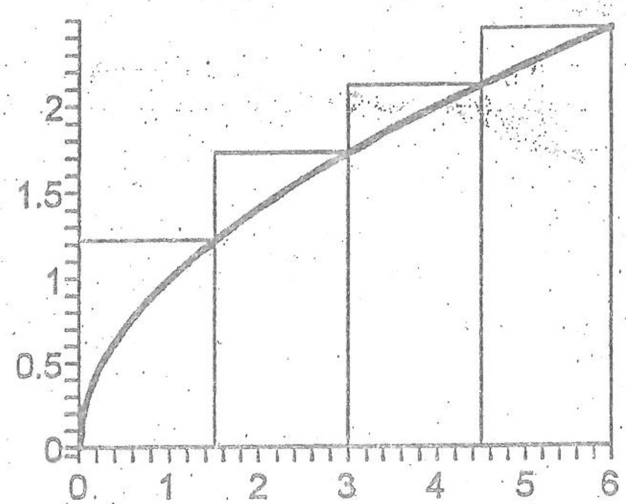
The *rightbox(f(x),x=a..b,n)* command graphs f(x) on [a,b], drawing n rectangles under f(x) to approximate the area

The *rightsum(f(x),x=a..b,n)* command find the exact sum of areas of rectangles from leftbox

Please note that the library *with(student)* must be open before you can use the command *rightbox*..

Example: Given the function $y = \sqrt{x}$, approximate a definite integral from x=0 to x=6.

```
> with (student):
  rightbox(sqrt(x), x=0..6, 4,color=MAGENTA);
  rightsum(sqrt(x), x=0..6,4);
```



$$\frac{3}{2}\sum_{i=1}^{4}\frac{1}{2}\sqrt{3}\,\sqrt{2}\,\sqrt{i}$$

As shown above rightsum does not produce the answer, however this can be accomplished by using the command *value* or *evalf* (recall that the procedure *evalf* gives the answer in decimal form).

```
> value(%);
```

$$\frac{3}{4}\sqrt{3}\,\sqrt{2}+\frac{3}{2}\sqrt{3}+\frac{9}{4}\sqrt{2}+\frac{3}{4}\sqrt{3}\,\sqrt{2}\,\sqrt{4}$$

```
> evalf(%);
```

$$11.29140865$$

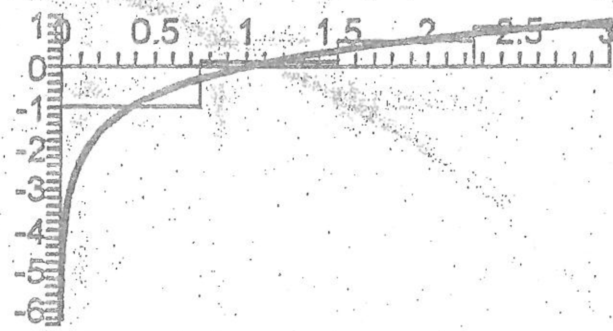
Integration by approximation

Middlebox (rectanglular boxes with the top middle of the boxes touching the graph)
The command *middlebox* will generate a plot of rectangular boxes used to approximate a definite integral. The height of each rectangle (box) is determined by the value of the function at the center of each interval. The formula for the corresponding numerical approximation to this integral is generated by the Maple procedure *middlesum*. Middlesum, therefore does the computation but does not produce the answer. This can be corrected by using the procedure *value* or *evalf* (recall that the procedure *evalf* gives the answer in decimal form).

Please note that the library *with(student)* must be open before you can use the command *middlesum*.

Example: Given the function y= ln(x), approximate a definite integral from x=0 to x=3.

```
> with(student):
  middlebox(ln(x), x=0..3, color=blue);
  middlesum(ln(x), x=0..3);
```



$$\frac{3}{4}\sum_{i=0}^{3}\ln\left(\frac{3}{4}i+\frac{3}{8}\right)$$

Remember to obtain the numerical value you must use the *value* or *evalf* command.

```
> value(%);
```

$$\frac{3}{4}\ln\left(\frac{3}{8}\right)+\frac{3}{4}\ln\left(\frac{9}{8}\right)+\frac{3}{4}\ln\left(\frac{15}{8}\right)+\frac{3}{4}\ln\left(\frac{21}{8}\right)$$

Recall that the procedure *evalf* gives the answer in decimal form.

```
> evalf(%);
```

$$0.5479825036$$

Lab Activity # 6:

a. Use the *leftbox*, *rightbox* and *middlebox* command to approximate a definite integral from $x=-\pi$ to $x=\pi$ for the function $\sin(x)$.

b. What do you notice about the answer when you use the different commands?

c. Explain why the answers are different or similar.

Section 8: Definite and Indefinite Integration

We can use the integration command to evaluate the integrals for most function, the input should be in the form of one of the following:

int(expr, x)	used to find the indefinite integral of 'expr'
Int(expr, x)	inert integral, not evaluated
int(expr, x=a..b, ...)	finds the definite integral of 'expr' between a and b
Int(expr, x=a..b, ...)	inert definite integral

- The function **int** computes an indefinite or definite integral of the expression expr with respect to the variable **x**. The name **integrate** is a synonym for **int**.

- Indefinite integration is performed if the second argument x is a name. Note that no constant of integration appears in the result. Definite integration is performed if the second argument is of the form x=a..b where a and b are the endpoints of the interval of integration.

- If Maple cannot find a closed form expression for the integral, the function call itself is returned.

- The capitalized function name Int is the inert version of the int function, which simply returns unevaluated. The pretty printer understands Int to be equivalent to int for printing purposes but formats the integral sign in black to visually distinguish the inert case. In this form, expr can actually be a procedure which can be integrated numerically.

Examples:

```
> int(x^2+2*x+12,x);
```

$$\frac{1}{3}x^3 + x^2 + 12\,x$$

Note: In the above results there is a misinterpretation of the integration, that is, Maple assume the integral to be a definite integral. Therefore, students must copy the above answer and add the constant as shown below.

```
> ans:=%+c;
```

$$ans := \frac{4}{3}x^3 + x^2 + 12\,x$$

Recall that the "%" sign is a call to the previous command..

Examples:

```
> Int(x^2+2*x+12,x);
```

$$\int x^2 + 2\,x + 12\,dx$$

The above shows the integral and the function, however it was not evaluated, to evaluate the integral you must use a lower case "i" in the "Int" command or you can use the value command.

Example 1: (using the lower case "i" in the integration command)

```
> int(3*x^2+7*x+12,x=0..2);
```

$$46$$

Example 2: (using the value command)

```
> Int(x^2+2*x+12,x);
```

$$\int x^2 + 2\,x + 12\,dx$$

```
> value(%)+c;
```

$$\frac{4}{3}x^3 + x^2 + 12x$$

The letter "c" was added to the command since Maple does not list the family of functions.

Double Integration

Doubleint(g, x, y)

Doubleint(g, x, y, Domain)

Doubleint(g, x = a..b , y = c..d)

Parameters

g – expression to be integrated

x, y – variables of integration

a, b, c, d – (optional) lower and upper bounds defining the range of integration

Domain – (optional) name identifying the region of integration

Example :

```
> with(student):
> Doubleint(x^4+2*x+12,x,x);
```

$$\int\int x^4 + 2x + 12 \, dx \, dx$$

If you have the region then you can enter the command as follows:

```
> Doubleint(x^4+2*x+12,x=-2..4,x=1..3);
```

$$\int_1^3 \int_{-2}^4 x^4 + 2x + 12 \, dx \, dx$$

```
> value(%);
```

$$\frac{2952}{5}$$

Recall that *value* gives the numerical answer of the computation

Recall that the command *evalf* gives the answer as a decimal, as shown below.

```
> evalf(%);
```

$$590.4000000$$

OR

The *int* command is used to compute single integrals as well as double and triple integrals. The command that computes the double integral $\int_a^b \int_c^d f(x, y)\, dy\, dx$ is *int(int(f(x,y),y=c..d),x=a..b)*. The definite integral is also numerically evaluated with the command *evalf(Int(Int(f(x,y),y=c..d,x=a..b))*.

Example: Evaluate the following integral $\int_1^2 \int_{1-y}^{\sqrt{y}} x\, y^2\, dx\, dy$.

```
> int(int(x*y^2,x=1-y..sqrt(y)),y=1..2);
```

$$\frac{163}{120}$$

Section 9: Change of variables

Calling Sequence

changevar(s, f)

changevar(s, f, u)

changevar(t, g, v)

- The changevar function performs a ``change of variables" for integrals, sums, or limits.
- The first argument is an equation defining the new variable in terms of the old variable. If more than two variables are involved, the new variable must be given as the third argument. The second argument is the expression to be rewritten and usually contains *Int*, *Sum*, or *Limit*.
- The change of variables may be implicitly defined (e.g. x^2+2 = 2*u^2).
- The unevaluated forms *Int*, *Limit*, and *Sum* should be used, rather than *int*, *limit*, and *sum*. They can be evaluated later by using the value command.
- Limited capabilities exist in connection with double and triple integrals. In this case, the equations defining the multivariate change of variables are given as a set, and the new variables are given in a list.
- The command *with(student,changevar)* allows the use of the abbreviated form of this command.

Example 1:

```
> with(student):
```

```
> changevar(cos(x)+1=u, Int((cos(x)+1)^3*sin(x), x), u);
```

$$\int -u^3\,du$$

In the above example the integration command with an upper case "I" is used, if I used a lower case "i", then the result is as seen below.

```
> changevar(cos(x)+1=u, int((cos(x)+1)^3*sin(x), x), u);
```

$$-\frac{1}{4}u^4$$

A close look will show that the above result is in the form of the variable "u", therefore this must be replaced with its original assignment.

```
> subs(u=cos(x)+1,%)+c;
```

$$-\frac{1}{4}(\cos(x)+1)^4+x^3$$

Example 2:

If you have the library *with(student)* open, then you do not need to open it again, on the other hand if not then you must open that library before you use the change of variable command.

```
> changevar(-x^2=u,Int(5*x*e^(-x^2),x));
```

$$\int -\frac{5}{2}e^u\,du$$

Remember, if you wish to have the numerical answer you can use either a lower case "i" in the integration command or use the *value* command.

```
> value(%);
```

$$-\frac{5\,e^u}{2\ln(e)}$$

```
> subs(u=-x^2,%)+c;
```

$$-\frac{5\,e^{(-x^2)}}{2\ln(e)}+x^3$$

Notice that the above answer can be simplified further. Take a close look at the denominator of the first term and you will be able to do this.

```
> sum(1/(4*n^2+8*n+3),n=1..infinity);
```

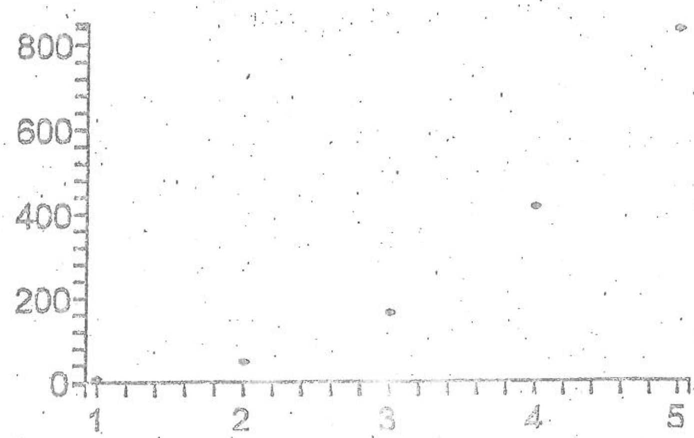
$$\frac{1}{6}$$

The command *seq([n,a(n)],n=1..k)* generates the sequence [1,a(1)], [2,a(2)], ..., [k,a(k)] of the function a(n) whose domain is the set of positive integers.

```
> a:=n->10^n/n!:
> pts:=[seq([n,a(n)],n=1..5)];
```

$$pts := \left[[1, 10], [2, 50], \left[3, \frac{500}{3}\right], \left[4, \frac{1250}{3}\right], \left[5, \frac{2500}{3}\right]\right]$$

```
> plot(pts,style=point);
```



Note: 1 is mapped onto a(1), 2 is mapped onto a(2) and so on.

Computing Power Series
Maple computes the power series expansion of a function *f(x)* about the point *x=a* up to order *n* with the command *series(f(x),x=a..n)*.
Note: The symbol *O*, appearing in the output indicates the terms that are omitted from the power series for *f(x)* about the point *x=a*.

Example: Find the first few terms of the power series cos(x) about the given point x=0.

```
> series(cos(x),x=0);
```

$$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 + O(x^6)$$

computes the terms of the power series for cos(x) about *x=0* to order 6; entering

```
> series(cos(x),x=0,7);
```

$$1 - \frac{1}{2}x^2 + \frac{1}{24}x^4 - \frac{1}{720}x^6 + O(x^7)$$

computes the terms of the power series for cos(x) about *x=0* to order 7

Section 11: Multivariable Calculus

Maple is useful in investigating functions involving more than one variable. Partial derivatives are easily calculated with the following commands:

diff(f(x,y),x)	computes the partial derivative with respect to x
diff(f(x,y),y)	computes the partial derivative with respect to y
diff(f(x,y),y,x)	computes the partial derivative with respect to y then with respect to x
diff(function,variable$n)	computes the n-th derivative of a function with respect to a variable

```
> h:=(x,y)->sin(x)^2*cos(y^2);
```

$$h := (x, y) \to \sin(x)^2 \cos(y^2)$$

```
> diff(h(x,y),x$2);
```

$$2 \cos(x)^2 \cos(y^2) - 2 \sin(x)^2 \cos(y^2)$$

Maple Quick Reference Commands

Basic Maple Commands

:	Causes the output resulting from a command to be suppressed
;	Ends a maple command
g : = (expr)	Assigns the function to a name or letter
f : = x ->(expr)	Defines *f* as a function of *x*.
D(f)(x)	Finds the derivative of a function f(x)
D(f)(a)	Finds the derivative of a function f(x) at the point a.
(D@@n)(f)(x)	Finds the n-th higher order derivative of the function f(x)
denom(expr)	Selects the denominator of a fraction
diff(expr, x)	Computes the derivative of the expression with respect to x
diff(f(x,y),x)	Computes the partial derivative with respect to x
diff(f(x,y),y)	Computes the partial derivative with respect to y
diff(f(x,y),y,x)	Differentiate with respect to y then with respect to x
diff(function, variable$n)	Computes the nth derivative of a function with respect to variable
display( )	Combines graphs of functions and points (requires *with(plots)*)
eval(expr,x = v)	Evaluates the expression at the point x=v
evalf(expr)	Numerically evaluates the given expression to the default number of digits
evalf(expr, n)	Numerically evaluates the given expression to *n* digits
expand(expr)	Expands the given expression
factor(expr)	Factors the given expression
fsolve(eqn)	Finds numerical (approximate) solution to equations
fsolve(eqn=0,a..b)	Finds numerical solution between a and b
ifactor(n)	Gives prime integer factorization for a given integer
implicitplot( )	Plots implicitly defined functions
int(expr, x)	Computes the integral of the expression with respect to *x*
int(int(f(x,y),y=c..d),x=a..b)	Used to find the double integral of a multi-variable function
leftbox(f(x),x=a..b,n)	Graphs f(x) on [a,b] drawing n rectangles under f(x) to approximate the area

leftsum(f(x),x=a..b,n)	Finds the exact sum of areas of rectangles from left box
lhs(eqn)	Selects the left hand side of an equation
limit(expr, x = v)	Computes the limit of the expression as *x* approaches *v*
numer()	Selects the numerator of a fraction
polarplot(expr, options)	Yields a 2-dimensional graph in polar coordinates
plot(expr,x=a..b)	Plots functions defined by an algebraic expression
plot({f(x),g(x)},x=a..b)	Plots the graph of f(x) and g(x) on the domain [a,b]
plot([x(t),y(t),t=a..b])	Plot of parametric curve x=x(t), y=y(t)
rationalize(expr)	Rationalizes the denominator of a given expression
restart	Clears maple's memory of all definitions
rightbox(f(x),x=a..b,n)	Graphs f(x) between a and b, putting n rectangles under graph of f(x)
rightsum(f(x),x=a..b,n)	Finds the exact sum of areas of rectangles from right box
rhs(eqn)	Selects the right hand side of an equation
seq([n,a(n)],n=1..k)	Generates the sequence [1,a(1)], [2,a(2)], …, [k,a(k)]
series(f(x),x=a..n)	Computes the power series expansion of a function f(x) about the about x=a up to order n
showtangent(f(x),x=c)	Displays the graph of f(x) and a tangent line at the point x=c
simplify(expr)	Simplifies the given expression
solve(eqn)	Find exact solutions to equations
subs(x = v, expr)	Substitutes the value v for x in the expression
sum(f(k),k=n..m)	Computes the sum of f(k) for the values of k from n to m
taylor(f,x=a,n)	Taylor polynomial of order n-1 at x=a for f
unapply(expr)	Returns an operator from the expression
unassign('var')	Clear the variable var
with()	Brings in additional libraries of functions

The standard constants

Maple Notation	Mathematical Notation
Pi	π Caution: Do not use "pi", capital "P" is required
exp(1)	e
I	$\sqrt{-1}$

Names of the standard functions

Maple Notation	Mathematical Notation
sqrt(x)	$\sqrt{x}$
abs(x)	$\|x\|$
exp(x)	e^x
ln(x)	natural log
log(x)	natural log, same as ln(x)
log[n](x)	$\log_n x$
sin(x)	$\sin x$
cos(x)	$\cos x$
tan(x)	$\tan x$
cot(x)	$\cot x$
sec(x)	$\sec x$
csc(x)	$\csc x$
arcsin(x)	$\arcsin x$
arccos(x)	arcos x
arctan(x)	$\arctan x$